Exercise sheet #5

Problem 1. Imagine a world where the force of interaction between two charges is:

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\left(1+\frac{r}{\lambda}\right)e^{(-r/\lambda)}\hat{r}$$

where λ is a constant with dimensions of length and $\lambda >> 0$ (i.e. it's really large so the correction to Coulomb's law given by the equation above is really small). Assuming that the principle of superposition still holds:

(a) What is the electric field of a charge distribution ρ? i.e. what is the equation that will replace $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r})}{r^2}\hat{r}dV$

(b) Does the electric field admit a scalar potential? Explain briefly your conclusion. Don't give a formal proof, rather a convincing argument.

(c) Find the potential ϕ for a point charge q (If your answer to (b) was no better go back and check it. Use ∞ as your reference point.

(d) Using the expressionf for $\vec{E}$ and ϕ you found. Show that for a point charge q at the origin:

$$\oint_S \vec{E}\cdot d\vec{a} + \frac{1}{\lambda^2}\int_V \phi dV = \frac{1}{\epsilon_0}q$$

where S is the surface and V the volume of any sphere centered at q.

(e) Show that this result generalizes to

$$\oint_S \vec{E}\cdot d\vec{a} + \frac{1}{\lambda^2}\int_V \phi dV = \frac{1}{\epsilon_0}Q_{enc}$$

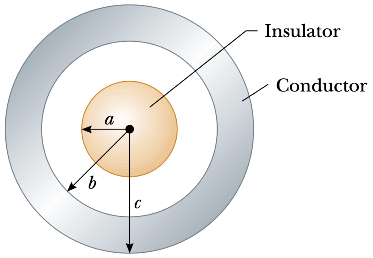
for any charge distribution.

(f) Show that, in contrast to our world, some of the charge on a conductor distributes itself **uniformly** over the volume, with the remainder on the surface. [Hint: First make an argument to show $\vec{E}$ is still zero, inside a conductor.]

Problem 2. A solid, insulating sphere of radius a has a uniform charge density ρ and a total charge Q. Concentric with this sphere is an uncharged, conducting hollow sphere whose inner and outer radii are b and c, as shown in the figure below.

(a) Find the magnitude of the electric field in the regions $r < a$, $a < r < b, b < r < c$, and $r > c$.

(b) Determine the induced charge per unit area on the inner and outer surfaces of the hollow sphere.



Problem 3. If a point charge is located outside a hollow conducting shell, there is an electric field outside, but no electric field inside. On the other hand, if a point charge is located inside a hollow conducting shell, there is an electric field both inside and outside (although the external field would be zero in the special case where the shell happened to have charge exactly equal and opposite to the point charge). The situation is therefore not symmetric with respect to inside and outside. Explain why this is the case, by considering where electric field lines can begin and end.

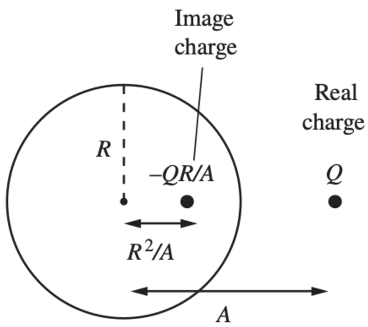
Problem 4. A point charge q is located between two parallel infinite conducting planes, a distance d from one and $l - d$ from the other. Where should image charges be located so that the electric field is everywhere perpendicular to the planes?

Problem 5. (a) A point charge $-q$ is located at $x = a$, and a point charge Q is located at $x = A$. Show that the locus of points with $\phi = 0$ is a circle in the xy plane (and hence a spherical shell in space).

(b) What is the relation among q, Q, a, and A so that the center of the circle is located at $x = 0$?

(c) Assuming that the relation you found in part (b) holds, what is the radius of the circle in terms of a and A ?

(d) Explain why the previous results imply the following statement: if a charge Q is externally located a distance $A > R$ from the center of a grounded conducting spherical shell with radius R, then the external field due to the shell is the same as the field of an image point charge $-q = -QR/A$ located a distance $a = R^2/A$ from the center of the shell (See Figure below). The total external field is the sum of this field plus the field from Q. (The internal field is zero, by the uniqueness theorem.)



Problem 6. Find the capacitance per unit length of two coaxial metal cylindrical tubes, of radii a and b.